**TTP95–36**
October 1995
hep-ph/9510406

# HADRONIC DECAYS OF EXCITED HEAVY QUARKONIA

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**Abstract**

We construct an effective Lagrangian for the hadronic decays of a heavy excited $s$-wave-spin-one quarkonium into its ground state. We show that reasonable fits to the measured invariant mass spectra in the $J/\psi$ and $\Upsilon$ systems can be obtained working in the chiral limit. The mass dependence of the various terms in the Lagrangian is discussed on the basis of a quark model.

# 1 Introduction

Bound state systems consisting of a heavy quark and a heavy antiquark are fairly well described in terms of wave function models, where the potential among the two heavy quarks is determined from fitting a phenomenological ansatz to the data [1]. In this way many static properties of these systems may be understood even quantitatively.

As far as decays of heavy quarkonia are concerned there has been also some theoretical progress recently [2, 3]. For inclusive decays in which the two heavy quarks annihilate an effective theory approach has been developed allowing for a systematic treatment of these processes. This method puts the description of this type of decays on a model independent basis.

The theoretical description of exclusive hadronic decays is in general still quite model dependent. However, for a large class of decay modes of excited quarkonia we may use chiral symmetry arguments to construct an effective Hamiltonian for decays such as $\Psi' \to \Psi\pi$ or $\Psi' \to \Psi\pi\pi$ where $\pi$ is a member of the Goldstone boson octet. This ansatz has been discussed already in the literature [4], however, without giving a systematic derivative expansion in the spirit of chiral perturbation theory.

Such an expansion may be performed starting from the infinite mass limit for the heavy quarkonium[1]. The momentum of the heavy quarkonium scales with the heavy quark masses and hence it may not be used as an expansion parameter. In order to define a systematic derivative expansion we decompose the momentum $p(p')$ of the heavy quarkonium in the initial (final) state as $p(p') = (m_Q + m_{\overline{Q}})v + k(k')$, where $k$ and $k'$ are small residual momenta of the order $\Lambda_{QCD}$, and $m_Q$ and $m_{\overline{Q}}$ are the masses of the heavy quark and antiquark respectively. The velocity $v$ is the velocity of the initial state quarkonium and hence the momenta of the final state heavy quarks will be of the order of the mass difference of the initial and final state quarkonia, which we shall assume to be small compared to the masses $m_Q$ and $m_{\overline{Q}}$.

In the spirit of chiral perturbation theory one may now formulate a derivative expansion, which after Fourier transformation becomes an expansion in the pion momenta and the residual momenta $k$ and $k'$. In the next section we shall use chiral symmetry to write down the leading as well as the chiral symmetry breaking terms in such an expansion. In section 3 we show that one may obtain a good fit to the data, total rates as well as invariant mass spectra of the two pions in the decay $\Psi' \to \Psi\pi\pi$. We shall compare the charmonia and the bottomonia and discuss the dependence of the parameters in the effective Lagrangian of the bottom and charm mass in section 4. Conclusions are given in section 5.

[1]The idea we are proposing here is in the same spirit as in [5], however, in [5] this method is applied to $\Phi$ decays.

## 2 Effective Lagrangian for Heavy Quarkonia Decays

We construct an effective Lagrangian for the decays of a heavy excited $s$-wave-spin-one quarkonium into the ground state, which is either a $J/\psi(1S)$ or an $\Upsilon(1S)$. Let $A_\mu$ be the field of the excited $s$-wave $1^-$ state and $B_\mu$ the one of $1^-$ ground state. We shall consider here only quarkonia where $m_Q = m_{\overline{Q}}$, hence we look at charmonia or bottomonia systems.

The momenta of the heavy quarkonia are split into a large piece which scales as $m_Q$ and a small residual part $k$ which depends only weakly on the heavy quark mass

$$
\begin{aligned}
p &= 2m_Q v + k \text{ for } B_\mu \\
p' &= 2m_Q v + k' \text{ for } A_\mu
\end{aligned}
$$

In coordinate space this is achieved by a phase redefinition of the fields

$$
\begin{aligned}
A_\mu(x) &= \exp(-i2m_Q v\cdot x) A_\mu^{(v)}(x) \\
B_\mu(x) &= \exp(-i2m_Q v\cdot x) B_\mu^{(v)}(x)
\end{aligned} \tag{1}
$$

such that the derivative acting on the fields with superscript $(v)$ is now $k$ (or $k'$) and small compared to the heavy mass. The fields $A^{(v)}$ and $B^{(v)}$ obey the equations of motion corresponding to static fields and are transverse with respect to the velocity vector $v$

$$
iv\cdot\partial A_\mu(x) = 0, \quad v^\mu A_\mu(x) = 0 \tag{2}
$$

$$
iv\cdot\partial B_\mu(x) = 0, \quad v^\mu B_\mu(x) = 0 \tag{3}
$$

We are interested in an effective Lagrangian for the hadronic decays of the type $A \to B\pi$ or $A \to B\pi\pi$, and we construct this Lagrangian using chiral symmetry. Higher order terms may be included by chiral perturbation theory, i.e. by a systematic expansion in the derivatives (these correspond to pion momenta or residual momenta for quarkonia) and in the light quark masses. The heavy quarkonia are singlets under the chiral symmetry, and hence the decays $A \to B\pi$ are forbidden in the chiral limit.

In the chiral limit only decays to an even number of pions are allowed, and the leading term in the derivative expansion obeying chiral symmetry consists of three terms

$$
\begin{aligned}
\mathcal{L}_0 = {} & g A_\mu^{(v)} B^{(v)\mu *}\ \mathrm{Tr}[(\partial_\nu U)(\partial^\nu U)^\dagger] + g_1 A_\mu^{(v)} B^{(v)\mu *}\ \mathrm{Tr}[(v\cdot\partial U)(v\cdot\partial U)^\dagger] \\
& + g_2 A_\mu^{(v)} B_\nu^{(v)*}\ \mathrm{Tr}[(\partial^\mu U)(\partial^\nu U)^\dagger + (\partial^\mu U)^\dagger(\partial^\nu U)] + \text{ h.c.}
\end{aligned} \tag{4}
$$

where $U$ is a unitary $3 \times 3$ matrix that contains the Goldstone fields

$$U = \exp\left(i\Phi/F_0\right) \qquad \Phi = \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta_8 \end{pmatrix} \tag{5}$$

and $F_0$ is the pion decay constant[2] in the chiral limit. In previous considerations the second term in (4) has been often omitted, but since a consistent definition of the derivative expansion forces us to introduce the vector $v$, the second term is a perfectly allowed one. We shall show in the next section that the magnitude of the coupling $g_1$ is not small compared to $g$, at least not for the case of charmonium, and that a reasonable fit to the invariant mass spectra of the pions may be obtained from (4).

The quark mass matrix breaks chiral symmetry, and the leading terms of this kind are

$$\begin{aligned} \mathcal{L}_{S.B.} = {} & g_3 A_\mu^{(v)} B^{(v)\mu *} \ \mathrm{Tr}[\mathcal{M}(U + U^\dagger - 2)] \\ & + i g' \epsilon^{\mu\nu\alpha\beta} \left[ v_\mu A_\nu^{(v)} \partial_\alpha B_\beta^{(v)*} - (\partial_\mu A_\nu^{(v)}) v_\alpha B_\beta^{(v)*} \right] \ \mathrm{Tr}[\mathcal{M}(U - U^\dagger)] \\ & + \text{h.c.} \end{aligned} \tag{6}$$

where

$$\mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{7}$$

The two operators proportional to the $\epsilon$ tensor are relevant for decays involving an odd number of pions. The relative sign of the these terms is fixed by reparameterization invariance [7]. An analogous formulation for the case $V' \to V\pi$ where $V', V$ are members of the lowest-lying vector meson nonet has been given recently by Jenkins, Manohar and Wise [5].

# 3 Fit to the Data

Starting from the Lagrangian given in (4) we fit the experimental data on the decays $\psi(2S) \to J/\psi\pi^+\pi^-$ and $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$. The amplitude has the form

$$\begin{aligned} \mathcal{A}(\Psi' \to \Psi\pi^+\pi^-) = -\frac{4}{F_0^2} \Big\{ & \left[ \frac{g}{2}(m_{\pi\pi}^2 - 2M_\pi^2) + g_1 E_{\pi^+} E_{\pi^-} \right] \epsilon_\Psi^* \cdot \epsilon_{\Psi'} \\ & + g_2 \left[ p_{\pi^+\mu} p_{\pi^-\nu} + p_{\pi^+\nu} p_{\pi^-\mu} \right] \epsilon_\Psi^{*\,\mu} \epsilon_{\Psi'}{}^{\nu} \Big\} \end{aligned} \tag{8}$$

[2] We use the convention in which the pion decay constant is $F_0 \simeq 93$ MeV.

| | $\psi(2S) \to J/\psi\pi^+\pi^-$ | $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$ | $\chi^2/d.f.$ |
|---|---|---|---|
| $g$ | $0.30 \pm 0.02$ | $0.25 \pm 0.02$ | 26.2 / 24 |
| $g_1$ | $-0.11 \pm 0.01$ | $-0.04 \pm 0.01$ | 26.3 / 19 |

Table 1: Values of the coupling constants obtained from a fit to the data as given in [9]. The errors quoted are purely experimental.

where $m_{\pi\pi} = (p_{\pi^+} + p_{\pi^-})$, $p_{\pi^+} = (E_{\pi^+}, \mathbf{p}_{\pi^+})$ and $\epsilon_\Psi$, $\epsilon_{\Psi'}$ are the polarization vectors of the heavy spin-one quarkonia. First we observe that the term proportional to $g_2$ describes the $d$-wave part of the pion system, which is found to be suppressed experimentally in the charmonium decay [8] as well as in the bottomonium decay [9] to a few percent. This observation is supported in addition by theoretical arguments, see next section. Thus we put $g_2 = 0$. The remaining coupling constants $g, g_1$ can be fitted to the total decay rate and the invariant mass spectra of the two pions. The data for the differential rate are taken from Fig.7 in [9]. For the total decay rate we used the PDG values [10]. Our results are listed in Table 1, and the fits are shown in Fig.1. We find that the values of $g$ in both systems are very close to each other, whereas the ratios $g_1/g$ differs by a factor of 2, namely

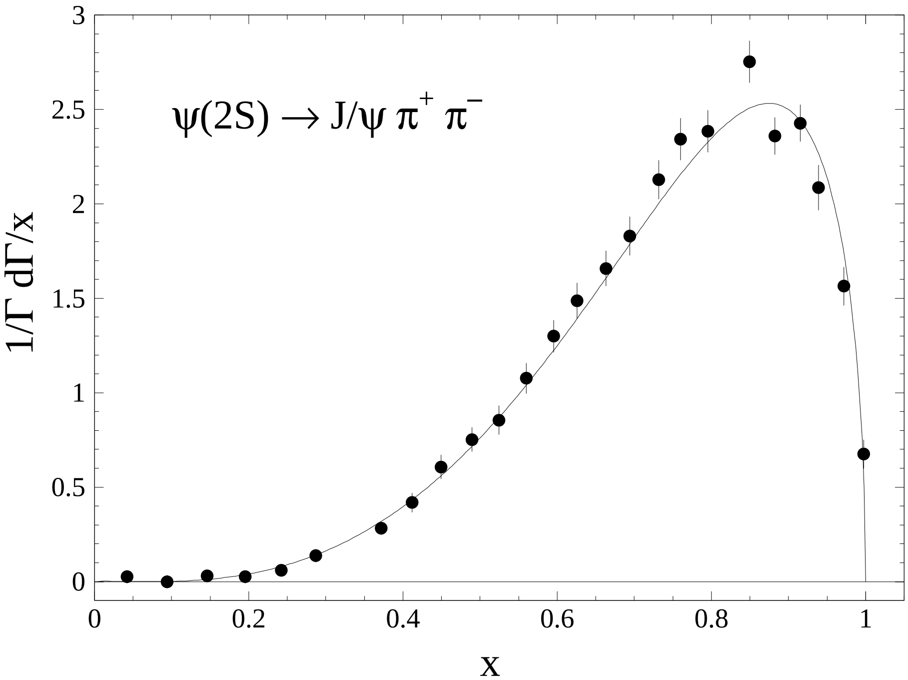
$$\left(\frac{g_1}{g}\right)_{c\bar{c}} = -0.35 \pm 0.03, \qquad \left(\frac{g_1}{g}\right)_{b\bar{b}} = -0.18 \pm 0.02 \tag{9}$$

With the inclusion of the symmetry breaking part (6) the situation does not change at all. If we consider the mass term as a perturbation to the operators relevant in the chiral limit, the best fit is again obtained for $g_3 = 0$[3]. The observation, that the data can be fitted accurately with operators allowed in the chiral limit, has been made by Novikov and Shifman [11]. To our knowledge, however, the rigorous description in the language of effective Lagrangians has not been presented before.

In the case of the decay amplitudes of $A \to B\pi$ where $\pi = \pi^0, \eta$ experimental values exist in the charmonium system, for bottomonia upper limits are known only. We consider the coupling $g'$ in the Lagrangian (6) with respect to the same decay channel in the different systems. The stronger limit is found in the $\eta$ decay channel

$$\left(\frac{g'_{b\bar{b}}}{g'_{c\bar{c}}}\right)_\eta < 0.6 \tag{10}$$

[3]In the case of charmonium $g_3$ is slightly different from zero, but still beyond the accuracy to which theresults are given.



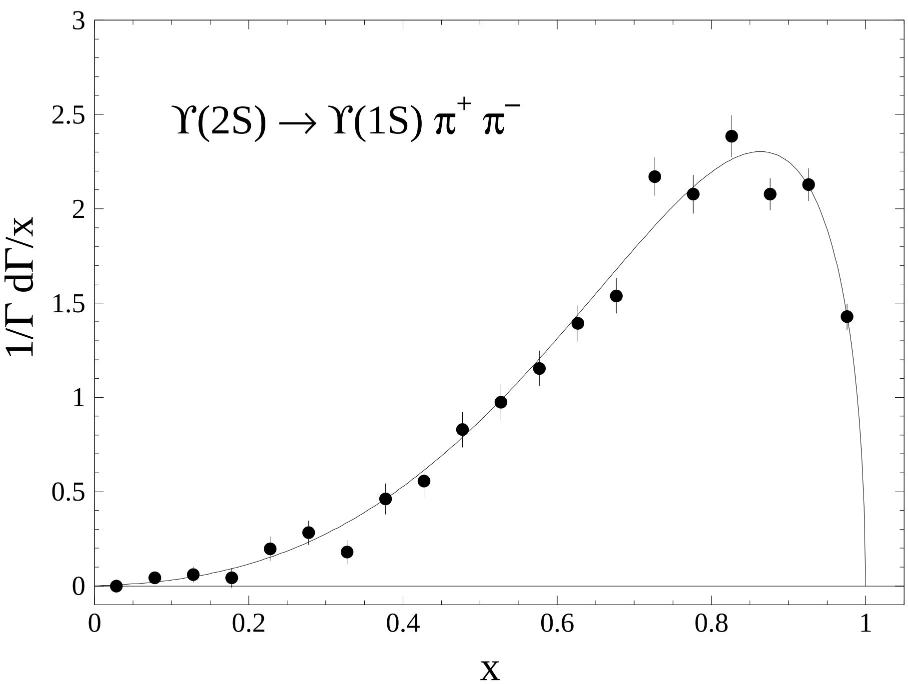




Figure 1: The curves show the fit to the invariant mass spectra of the two pions with $x = (m_{\pi\pi} - 2M_\pi)/(M_{2S} - M_{1S} - 2M_\pi)$ and $m_{\pi\pi} = (p_{\pi^+} + p_{\pi^-})$. The experimental data are taken from [9].

# 4 Discussion of the Results

In this section we shall discuss our results, in particular the mass dependence of the coefficients $g$, $g_1$ and $g'$. We shall try to achieve an at least qualitative understanding by representing the coefficients in the Lagrangian by matrix elements of heavy quark operators. Hence we consider a model in which the heavy quarks and the light pseudoscalar mesons are the relevant degrees of freedom; in this respect this resembles the chiral quark model of Georgi and Manohar [6].

First of all, this model easily explains the fact that the pion-$d$-wave contribution is suppressed, since this at least requires a dimension-6-operator for the heavy quarks, which would be contained in the operator

$$\mathcal{L}_{d-wave} = \frac{a_2}{\Lambda^3}(\bar{Q}_v\gamma_\mu Q_v)(\bar{Q}_v\gamma_\nu Q_v)\mathrm{Tr}\ \left[(\partial^\mu U)(\partial^\nu U^\dagger) + (\partial^\nu U)(\partial^\mu U^\dagger)\right] \tag{11}$$

while the pion-$s$-wave component may be represented by a dimension-3-operator for the heavy quarks

$$\mathcal{L}_{s-wave} = a_0(\bar{Q}_v Q_v)\mathrm{Tr}\ \left[(\partial^\mu U)(\partial_\mu U^\dagger)\right] \tag{12}$$

Here $Q_v$ are the operators of heavy quarks which have been rescaled with a phase according to

$$Q_v = \exp(imvx)Q$$

and the scale $\Lambda$ is at least of the order of the chiral symmetry breaking scale or maybe even the heavy quark mass; in the latter case the $d$-wave contribution to this decay would be more strongly suppressed in the $\Upsilon$ decays as compared to the $J/\psi$ transitions.

We have argued that the $d$-wave contribution is suppressed and hence we have only two operators left in the chiral limit. Both operators contribute to the $s$-wave of the decays $\psi(2S) \to J/\psi\pi\pi$ and $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$. Employing the model discussed above, we have the corresponding two operators on the quark level

$$\begin{aligned}\mathcal{L}_{quark} &= a_0(\bar{Q}_v Q_v)\mathrm{Tr}\ \left[(\partial^\mu U)(\partial_\mu U^\dagger)\right] \\ &+ b_0(\bar{Q}_v Q_v)\mathrm{Tr}\ \left[(v\cdot\partial U)(v\cdot\partial U^\dagger)\right]\end{aligned} \tag{13}$$

The coupling constants $g$ and $g_1$ are related to the couplings in the quark Lagrangian $\mathcal{L}_{quark}$ through the relations

$$g\langle\Psi|A^\mu B^*_\mu|\Psi'\rangle = g\epsilon^*_\Psi\cdot\epsilon_{\Psi'} = a_0\langle\Psi|\bar{Q}_v Q_v|\Psi'\rangle \tag{14}$$

$$g_1\langle\Psi|A^\mu B^*_\mu|\Psi'\rangle = g_1\epsilon^*_\Psi\cdot\epsilon_{\Psi'} = b_0\langle\Psi|\bar{Q}_v Q_v|\Psi'\rangle \tag{15}$$

From the combined chiral and heavy mass expansion one would expect that both coupling constants $a_0$ and $b_0$ scale in the same way with the heavy quark mass. As

far as the mass dependence of $g$ and $g_1$ is concerned, one has to take into account the mass dependence of the matrix element $\langle\Psi|\bar{Q}_v Q_v|\Psi'\rangle$. For the operators itself there exists a heavy mass limit, while the mass dependence of the states may not be accessed within the framework of the $1/m_Q$ expansion [3]. To this end we have to rely on simple dimensionality arguments. Assuming that the states are normalized to unity we may write the matrix elements as

$$\langle\Psi|\bar{Q}_v Q_v|\Psi'\rangle \propto \Lambda^3 \tag{16}$$

where $\Lambda$ is a scale related to the binding of the heavy quarkonia. Here we may consider two extreme cases. The first one is the purely coulombic scenario, in which the parameter $\Lambda$ is the heavy mass times the fine structure constant

$$\Lambda = \alpha_s(m_Q) m_Q \tag{17}$$

and therefore one would expect a strong scaling with the heavy mass. This is, however, not supported by the energy differences in the known quarkonia, where e.g. the level spacing $M_{2S} - M_{1S}$ is practically the same in the $J/\psi$ system and the $\Upsilon$ system. So we shall argue that a scenario in which $\Lambda$ is independent of the mass of the heavy quark is realistic, and consequently we would expect that $g$ and $g_1$ should scale in the same way as the parameters $a_0$ and $b_0$ in the quark level Lagrangian $\mathcal{L}_{quark}$. Since $g_{b\bar{b}}/g_{c\bar{c}}$ is practically unity, we may assume that both couplings are of order one in the heavy mass expansion. This in turn would lead to the conclusion that $g_1/g$ should not scale as a power of the heavy mass. However, the coupling $g_1$ is larger in the $J/\psi$ system by a factor between two or three compared to the $\Upsilon$ system. The only consistent interpretation of this factor is that

$$\frac{g_{1\,b\bar{b}}}{g_{1\,c\bar{c}}} = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{\kappa}, \qquad \kappa \simeq 2-3 \tag{18}$$

Consequently the second term of our effective Lagrangian must be governed at the level of QCD by an operator with a sufficiently large anomalous dimension.

Let us finally also consider the chiral symmetry breaking contribution relevant for the decays $\psi(2S) \to J/\psi\pi$ and $\Upsilon(2S) \to \Upsilon(1S)\pi$. Interpreting this transition again in terms of a dimension-3 operator for the heavy quarks one writes

$$\mathcal{L}'_{quark} = a'_0(\bar{Q}_v\gamma_5 Q_v)\mathrm{Tr}\ \left[(U - U^\dagger)\right] \tag{19}$$

The leading term in the heavy mass expansion will vanish for this operator, since

$$\bar{Q}_v\gamma_5 Q_v = \mathcal{O}(1/m_Q)$$

and hence one expects the coefficients $g'$ to scale as

$$\frac{g'_{b\bar{b}}}{g'_{c\bar{c}}} = \frac{m_c}{m_b} \simeq 0.3 \tag{20}$$

assuming again that the mass dependence of the matrix elements is weak.

# 5 Conclusions

We have formulated an effective theory approach for the hadronic decays of heavy quarkonia based on the chiral limit for the light degrees of freedom and on the $1/m_Q$ expansion for the heavy ones. The leading term in both the inverse of the heavy quark mass and the inverse of the chiral symmetry breaking scale consists of two terms multiplied by coupling constants which we have fitted from experiment. The fit to the invariant mass spectrum of the two pions in the decays $\psi(2S) \to J/\psi\pi^+\pi^-$ and $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$ shown in Fig.1 is quite satisfactory.

We also have discussed the mass dependence of the various terms in the Lagrangian which unfortunately may not be considered entirely in the effective theory. We model the leading terms of the effective Lagrangian by switching to a description in which the relevant degrees of freedom are pions and heavy quarks. This model resembles the chiral quark model as proposed by Georgi and Manohar [6]. In this picture the coupling constants of the quarkonia-pions effective Lagrangian may be reexpressed in terms of the ones of the underlying quark Lagrangian and certain matrix elements of heavy quark operators between heavy quarkonia states. The mass dependence of the matrix elements may not be evaluated from chiral and heavy quark symmetry and thus requires additional input. Using a specific assumption on the mass dependence of these matrix elements we arrive at definite predictions for the ratios $g_{b\bar{b}}/g_{c\bar{c}}$ and $g'_{b\bar{b}}/g'_{c\bar{c}}$. The predictions are consistent with what is found experimentally, although for $g'_{b\bar{b}}/g'_{c\bar{c}}$ only an experimental bound exists and more data is needed to confirm or falsify our values.

# Acknowledgements

R.U. thanks H.Leutwyler for discussions and H.Genz for providing unpublished notes.

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